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Space-Time Structure of Jet Hadronization

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ABSTRACT

The space-time development of jet hadronization is investigated in two-dimensional quantum electrodynamics. It is found that the characteristic space-time scale of jet hadronization is considerably shorter than the one given by approximately free propagation of quarks.

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Recently a great deal of progress has been made in understanding inclusive jet phenomena.[1] However, there is still lacking an understanding of the final stage of jet evolution i.e., jet hadronization. Aside from an interesting attempt by Amati and Veneziano,[2] we know very little about the hadronization.

In this note, we wish to address the question of jet hadronization in solvable two-dimensional (1-space and 1-time) gauge theories. These theories seem to provide us a unique place to investigate the interplay between hard and soft processes.

One of the most important aspects of jet hadronization may be its time scale. Therefore we first develop, in a systematic way, a formalism which enables us to describe the space-time structure of jet hadronization. Secondly, we will discuss two-dimensional gauge theories.

Following Carruthers and Zachariasen[3] we introduce a field theoretic version of Wigner's phase space distribution[4] in quantum mechanics,

$$\tilde{F}(p,R) = \int d^2r \; e^{ipr} (\Box + m^2)_{R-r/2} \; \langle\Phi|\phi(R-\tfrac{1}{2}r)\;\phi(R+\tfrac{1}{2}r)|\Phi\rangle \times (\Box + m^2)_{R+r/2} \; . \qquad (1)$$

Here $|\Phi\rangle$ is a normalized Heisenberg "in" state, and ϕ is the Heisenberg operator for a hadron with mass m. We deal with the amputated quantity since it is directly related to an observable quantity;

$$\frac{1}{\sigma}\,(2\pi)2\omega_p\,\frac{d\sigma}{dp} \equiv \langle\Phi|a^{+}_{out}(p)a_{out}(p)|\Phi\rangle \;.$$

$$= \int d^2R\;\tilde{F}(p,R)\Big|_{p^2=m^2} \;. \qquad (2)$$

If we impose the mass shell condition, it is clear from the above equation that $\tilde{F}(p,R)$ contains the information about which space-time region dominantly contributes in producing the hadron of momentum p. Although we restrict ourselves to the production of spinless hadrons in two-dimensional space-time, the generalization of our formalism to more realistic cases is straightforward.

Let us examine how the formalism works in massless quantum electrodynamics in 1-space and 1-time dimension (Schwinger model).[5,6] As in Ref. 6 we take the classical external source approximation of receding quark and antiquark in e^+e^- annihilation (in this note, we use the terms quark and gluon instead of electron and photon):

$$j_0^{ext} = g\delta(x-t),\; j_1^{ext} = g\delta(x-t). \qquad \text{for } x>0$$

$$j_0^{ext} = -g\delta(x+t),\; j_1^{ext} = g\delta(x+t). \qquad \text{for } x<0 \qquad (3)$$

Using the formulas in Ref. 3 $\tilde{F}(p,R)$ can readily be calculated as

$$\tilde{F}(p,R) = \lim_{\varepsilon \to 0} \frac{8(gm)^2}{p_+ p_-} \sin(p_+ R_-) \sin(p_- R_+)$$

$$\times \theta(R_+)\theta(R_-)\, e^{-\varepsilon R_+}\, e^{-\varepsilon R_-} \quad , \qquad (4)$$

where $p_\pm = (p_0 \pm p_1)$ etc. The above expression shows that the dominant contribution comes from the region $p_+ R_- \lesssim 1$ and $p_- R_+ \lesssim 1$.[7] Thus we reproduce the well-known hyperbola $R_+ R_- \lesssim m^{-2}$ [8] which is demonstrated by Casher, Kogut, and Susskind,[6] who used a purely classical argument.

Let us proceed to a quantum treatment of the same problem. In particular, we want to go beyond the classical external source approximation of separating quarks. In order to do this, we propose to use the modified phase space distribution

$$\tilde{F}(p,R) = \frac{1}{2i} \text{ disc} \int d^2x\, e^{iQx} \int d^2r\, e^{ipr}$$

$$\times (\Box + m^2)_{R-r/2} \langle 0 | T\ s(x) \phi(R - \tfrac{r}{2}) \phi(R + \tfrac{r}{2}) s(0) | 0 \rangle$$

$$\times (\Box + m^2)_{R+r/2} \quad , \qquad (5)$$

where $s(x)$ is a current (indices abbreviated) coupled to a time-like photon with mass Q^2. The prescript "disc" means to take the discontinuity in the channel appropriate for the reaction $e^+e^- \to$ hadron (p) + anything. Notice that $\tilde{F}(p,R)$ in (5) also satisfies the relation (2) apart from some multiplicative factor.

In the original definition (1),[3] $|\Phi\rangle$ cannot be a momentum eigenstate because, if so, translational invariance then requires $\tilde{F}(p,R)$ to be independent of R. This difficulty can be avoided if the reactions are induced by a local current. The e^+e^- annihilation is an ideal place for this since the space-time point x in (5) is kinematically confined in the region $\sim 1/\sqrt{Q^2}$. Thus we expect that $\tilde{F}(p,R)$ in (5) provides information on the space-time development of jet hadronization within a (spatio-temporal) resolution $1/\sqrt{Q^2}$.

In this paper we restrict ourselves to the scalar current $s(x)=\bar{\psi}(x)\psi(x)$ since it bears a closer resemblance to the electromagnetic current in four dimensions than does the vector current.[6] Use of the pseudoscalar current would not affect our qualitative conclusions.

Since the model is exactly soluble[5] it is straightforward to calculate the phase space distribution. In this model $\phi(x)$ in (5) denotes the massive boson field which appears in the physical spectrum of the theory. We have

$$\tilde{F}(p,R) = C\, e^{2i(Q-p)R}\, e^{4\pi^2 \Delta^{(+)}(m,2R)} , \tag{6}$$

where C is a constant containing Euler's constant and

$$\Delta^{(+)}(m,x) = \int \frac{dk}{(2\pi)2\omega_k}\, e^{-ikx} . \tag{7}$$

This result is entirely different from the one with the external source approximation. The dominant contribution comes from $R_\mu R^\mu \lesssim 1/(Q-p)^2 \simeq O(1/Q^2)$. The last equality follows from the fact the missing mass squared is in general of the order of $\sim Q^2$ except for a restricted kinematical region.[9]

Some readers may be doubtful about such a drastic change in the result. However the result (6) appears quite natural if we look at the diagram of scalar photon vacuum polarization dipicted in Fig. 1. In terms of hadronic variables (left-hand-side of Fig. 1) the process scalar photon $\rightarrow$ hadrons occurs instantaneously. The phase space distribution (6) describes this fact within a limited resolution $\sim 1/\sqrt{Q^2}$.

Let us examine the same process in terms of quark-gluon variables. (Right-hand-side of Fig. 1.) First we show that the result in terms of hadronic variables can be reproduced by summing the perturbation servies in quark-gluon basis. Since the quantity under discussion $\langle 0|T\, s(x)\, s(0)\, |0\rangle$ is gauge invariant, we can choose the light-cone gauge without loss of generality. In this gauge, the upper and lower parts of the quark line in Fig. 1 do not communicate with each other nor do the diagrams with radiative corrections on both quark lines survive because of the γ-matrix algebra. Then, using Stamatescu and Wu's result[10] on the quark propagator, we can immediately verify the above conclusions.

We note that the same process looks quite different in hadronic and quark-gluon bases. In the latter, a quark radiates successive gluons with vacuum polarization corrections in contrast to the instanteous production in the former. Whereas the confinement force is already operative in a very short space-time region $R^2 \sim 1/Q^2$, the system can be described by the quark-gluon language to a longer time scale. Since quarks and gluons are not in the physical spectrum of the theory, it is, in general, a difficult question how to evaluate this time scale.

We argue, however, that this time scale can be estimated by the following consideration.[6] Since the theory is asymptotically free the short-time behavior of e^+e^- final states should be described by almost free quarks. This can be seen in view of the expression corresponding to Fig. 1,

$$\langle 0|T\ s(x)s(0)|0\rangle = \frac{1}{2\pi^2 x^2}\, e^{-4i\pi\Delta_F(0,x)}\, e^{4i\pi\Delta_F(m,x)} \ , \qquad (8)$$

where $\Delta_F(\mu,x)$ is the Feynman propagator of mass μ quanta. If $x^2 \lesssim m^{-2}$ the exponent $\Delta_F(0,x)-\Delta_F(m,x)$ tends to zero and we are left with free fermion singularity $1/x^2$. Then we obtain the space-time scale $x^2 \simeq m^{-2}$ beyond which the free quark description breaks down.

Of course we could describe the system entirely by hadronic variables since the logarithmic singularity in $\Delta_F(0,x)$ precisely cancels the free quark singularity x^{-2} in (8). Thus in the region $Q^{-2} \lesssim x^2 \lesssim m^{-2}$ we can describe the system by either hadronic or quark-gluon languages.[11]

So far we have confined ourselves to the massless Schwinger model. We now try to examine the stability under a quark mass. Since the massive Schwinger model is not soluble (at present), we restrict ourselves to the case of small quark mass $\mu \ll g$ and rely upon mass perturbation theory. We consider the bosonized form of the theory and choose the perturbative vacuum $|n=0\rangle$ rather than the θ-vacuum.[12] This is because the result (8) of the original Schwinger solution and of the perturbation series summation corresponds to the choice of the n=0 vacuum.[13] This restriction does not seem to affect our qualitative conclusions.

The mass has the potential to invalidate our conclusion about the short time ($\sim 1/Q$) hadronization in the massless model. For instance the diagram dipicted in Fig. 2 contributes to second order in μ. However the calculated result of the phase space distribution is

$$\tilde{F}(p.R) = \left(\frac{K}{2}\right)^2\left(\frac{\mu K}{2}\right)^2 \sum{}' \, (4\pi\delta\delta') \prod_{i=1}^{3} \int \frac{d^2k_i}{(2\pi)^2} \int d^2\xi$$

$$\times\, e^{i(Q-k_1-k_2)R}\, e^{i\left(\frac{Q-k_1-k_2}{2} - k_3 - p\right)\xi}$$

$$\times\, e^{-4\pi^2\varepsilon\varepsilon'\Delta^{(+)}(m,\xi)}\, e^{-4\pi\varepsilon'\delta' i\Delta_F\left(m,R-\frac{\xi}{2}\right)}$$

$$\times\, S^{(+)}_{\varepsilon\varepsilon'}(k_1^2) S^{(+)}_{\varepsilon'\delta}(k_2^2) S^{(+)}_{\varepsilon\delta'}(k_3^2) S_{\varepsilon\delta}\left[(Q-k_1-k_3^2)\right]. \qquad (9)$$

Here Σ' means the summation over $\varepsilon,\varepsilon',\delta$ and $\delta' = \pm 1$ under the constraint $\varepsilon+\varepsilon'+\delta+\delta'=0$ and K is a numerical constant containing Euler's constant. The function S is defined by

$$S_{\varepsilon\delta}(k^2) = \int d^2x \; e^{-ikx} \langle 0|T{:}e^{2\sqrt{\pi}i\varepsilon\phi(x)}{:}{:}e^{2\sqrt{\pi}i\delta\phi(0)}{:}|0\rangle \; , \qquad (10)$$

and $S^{(+)}_{\varepsilon\delta} = \mathrm{Im} S_{\varepsilon\delta}$. From (9) it is clear that $R^2 \lesssim O(1/Q^2)$ except for a restricted kinematical region. We have also checked the stability of the result by including the remaining diagrams up to second order in μ.

It is more interesting to examine the problem in two-dimensional quantum chromodynamics. The $1/N_c$ expansion, however, is inappropriate to investigate the space-time structure since directly produced resonances live an infinitely long time if $N_c \to \infty$.

From studies of two-dimensional QED we are led to the apparently puzzling conclusion that the space-time scale of (approximately) free propagation of quarks is larger than the hadronization space-time scale. This conflicts with classical intuition[14] but can be consistent with quantum mechanics; it means that between these two time scales the overlapping of the hadron wave functions is so important as to allow the approximately free quark description of the hadron system. We emphasize the existence of (at least) one consistent theory which satisfies simultaneously the asymptotic freedom and "hard" confinement.

One may ask which aspect of our result survives for the real four-dimensional world. We are tempted to speculate that the short-space-time hadronization in two dimensions survives in the form of short-space-time color screening in four dimensions.[15] Although the screening and the hadronization may be the same thing in two dimensions, it probably is not in four dimensions since there may be certain reshuffling processes because of the physical degrees of freedom of the gluon. If the hierarchy of the two different time scales (i.e., one of hadronization and one of free quark propagation) survives in spite of this difference between two and four dimensions, it enables us to understand the duality[16] between free-quark and potential-model (hadronic) descriptions of e^+e^- final states.

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FOOTNOTES

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probabilistic interpretation. However one can discuss the problem which space-time region dominantly contributes in a sense of Riemann-Lebesgue's lemma.

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9. It is doubtful if $\tilde{F}(p,R)$ has a sensible meaning for the two-body exclusive process. Since particle's momenta are kinematically determined in this process the definite information on space-time seems to contradict with uncertainty principle.

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11. In reaching to this conclusion, we owe very much to the discussion with J.D. Bjorken and to Ref. 6.

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FIGURE CAPTIONS

Fig. 1 Schematic illustraction of current correlation function (8). The dashed, solid, and wavy lines indicate massive bosons, quarks, and gluons, respectively.

Fig. 2 Illustration of a contribution to $\tilde{F}(p,R)$ in second order of quark mass μ.

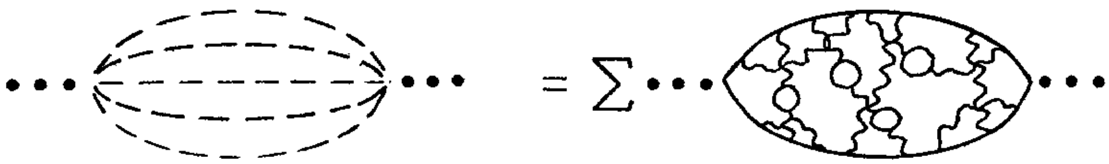

Fig. 1

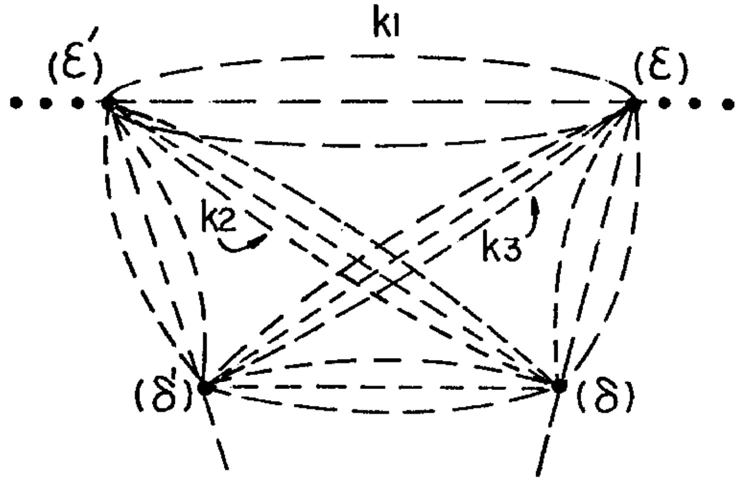

Fig. 2